# Adoption of AI Technology and Multi-Product Duopoly

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**Abstract**

Our work investigates the adoption of AI technology by firms in a duopoly competitive environment with multiple products, particularly focusing on the optimal pricing strategy for AI companies when their technology could significantly impact the downstream market. AI technology enhances product quality, influencing consumer willingness to pay, but also increases unit costs for users and affects competitive dynamics across market segments. We explore a scenario where a technological leader and follower engage in multi-product duopolistic competition, analyzing the effects of AI adoption on market shares and competitive behavior in segmented markets. The study reveals that AI adoption can either increase or decrease baseline consumption, depending on the AI usage fee, and that a single company's adoption of AI can lead to a competitive advantage. The optimal pricing strategy for AI services is examined in the context of downstream firms' adoption and production strategies, highlighting a trade-off between high AI service prices and widespread adoption among downstream companies. This research contributes to the AI adoption literature by offering a nuanced understanding of how AI technology intersects with multi-product competition and pricing strategies.

# 1 Introduction

AI technology revolutionized traditional equipment and facilities, enhancing their functionality in unprecedented ways. Take, for instance, the burgeoning Driver Assistance System (DAS), which

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assists drivers in mundane tasks, thus significantly lightening the load of routine navigation. This trend is gaining momentum as more auto manufacturers are incorporating DAS into their vehicles, making these upgraded systems increasingly appealing to customers who once prioritized conventional vehicles for their robust engines, superior handling, and luxurious interiors.

However, despite the transformative potential of AI, not all companies take prompt actions to embrace this evolution. One notable reason is that breakthroughs in AI often stems from companies soly dedicated to its development rather than its application. This means that for AI technology to penetrate traditional industries, these development-focused companies must successfully advocate their services to firms operating in those sectors — an endeavor that doesn't always yield positive returns. As a result, many traditional companies are hesitant to adopt AI-driven improvements, and even companies with advanced technology sometimes exhibit reluctance in integrating AI into their product design.

With these considerations in mind, we aim to explore the following question: What pricing strategy should AI company adopt when their innovative technology has the potential to profoundly influence the downstream market?

AI technology has a profound impact on the downstream market, manifesting in three key aspects. First, it elevates the quality of products or services when integrated, enhancing consumers' willingness to pay for them. Second, it increases the unit cost for manufacturers, reflecting the charges imposed by the AI company for its services. Third, it indirectly affects competition among downstream firms operating in segmented markets. Companies with diverse product lines aimed at different segments may leverage AI to bolster their market shares.

In our scenario, we have a traditional industry where two players, a technological leader and a follower, engage in multi-product duopolistic competition. This competition is framed within a context of consumer behavior where consumers initially opt for baseline products from suppliers and may later choose to upgrade to higher tier offerings. When both companies adopt AI technology, the enhancements it brings cancel out in the upgrade markets. Consequently, the consumer segments purchasing high-end products remain stable, while baseline consumption may fluctuate based on the AI usage fee. However, if only one company adopts AI, its technological edge in AI leads to increased competitiveness in the upgrade market due to lower upgrading costs. This AI-adopting company will display more aggressive behavior compared to its competitors in these specific market segments.

When considering the optimal pricing strategy for its AI service, the AI company must carefully evaluate the adoption and production strategies of downstream firms. A low usage price suggests that adopting AI becomes the prevailing equilibrium strategy. Any company failing to keep pace with AI adoption finds itself at a distinct disadvantage. However, as the usage price rises, the equilibrium shifts to a scenario where at most one company adopts AI. In such cases, a company may choose to sacrifice market shares rather than incur the additional cost of using AI. It is noteworthy that, in such an asymmetric situation, the profit for the AI user (non-user) is negatively (positively) related to the AI usage fee. Ultimately, if the price becomes excessively high, both companies may opt out of using AI altogether. The AI company would never set a price that drives away all customers. The primary trade-off lies between setting a high price for AI service and encouraging more downstream companies to adopt AI service and increase their production volume.

The remainder of the paper is organized as follows. In Section 2, we briefly discuss related literature in multi-product duopolistic market and in AI development. In Section 3, we lay out the theoretical framework for subsequent analyses. In Section 4, we present basic results that include characterization of firms' profits and competition strategies in the downstream market. In Section 5, we propose a numerical example to illustrate the possibility that at AI company's optimal pricing, it is the technological follower in the traditional industry that would definitely adopt AI, whereas the technological leader randomizes.

# 2 Literature Review

The literature on multi-product competition and its implications for pricing, product quality, and market structure is rich and multifaceted, with contributions from a range of scholars who examine these issues from various theoretical and empirical perspectives, which provides a comprehensive overview of the dynamics of multi-product firms in competitive markets.

Champsaur and Rochet (1989) delve into the strategic behavior of multi-product duopolists, this body of literature expands to encompass the complexities of multi-product pricing (Armstrong and Vickers, 2018), the role of flexible manufacturing in global economy (Eckel and Neary, 2010), and the optimization of multi-product pricing under differentiated price sensitivities (Gallego and Wang, 2014). These studies lay the groundwork for understanding how multi-product firms navigate and shape the competitive landscapes in which they operate.

The series works of Johnson, Myatt and Rhodes further enrich this field by exploring a variety of dimensions related to multi-product firms. Johnson and Myatt (2003) present a model of multi-product monopoly and duopoly using a general "upgrades" approach. The authors delve into the strategic dimensions of quality competition among firms offering multiple products. Their study provides a theoretical framework to understand how firms manage their product lines in competitive markets, particularly focusing on the deployment of fighting brands and the decision to prune product lines. They build upon the existing literature on product differentiation and competition by introducing a model that captures the complexities of multi-product strategies in a competitive environment. Their analysis highlights the conditions under which a firm might introduce lower-quality, lower-priced products to fend off competitors, or conversely, reduce its product offerings to concentrate on higher-quality, higher-margin products. Johnson and Myatt (2006a) delve into the dynamics of multi-product Cournot oligopolies, offering insights into how firms in oligopolistic markets manage and strategize their product lines when faced with competitive pressures. Johnson and Myatt (2006b) explore the economics of advertising, marketing, and product design, emphasizing the strategic role of these elements in differentiating products and influencing consumer preferences. Johnson and Myatt (2015) further contribute to the understanding of how pricing strategies are employed across different product lines, incorporating the effects of competition, cost structures, and market segmentation on pricing decisions. Johnson (2017) focus on consumer behavior and retail strategies, highlighting how retail environments are designed to maximize consumer spending and how this interacts with competitive dynamics in the retail sector. Johnson and

Myatt (2018) investigate the determinants of product lines. This work provides a deeper dive into the strategic decisions behind the breadth and depth of product lines offered by firms, considering cost asymmetries and the desire to soften competition. Johnson and Rhodes (2021) examine multi-product mergers and quality competition. This research extends the discourse on how mergers between firms with multiple product lines affect market competition, product quality, and consumer welfare, offering new perspectives on the implications of corporate consolidation in competitive markets.

Manova and Yu (2017) contribute to the discussion by examining how multi-product firms navigate product quality decisions within the international trade context, highlighting the strategic considerations firms face when managing diverse product lines across different markets. Nocke and Schutz (2018) introduce an aggregative games approach to the study of multi-product oligopoly, providing a novel methodological framework to analyze market outcomes in multi-product settings.

The literature on AI adoption delves into various aspects, including its potential disruptive impacts on employment and industries. Agrawal et al. (2021) provide a foundational understanding by discussing system-wide changes due to AI integration. Chatterjee et al. (2021) focus on manufacturing and production firms, employing an integrated TAM-TOE model to elucidate the factors influencing AI adoption in these specific sectors, offering insights into the technological, organizational, and environmental determinants of AI integration. Gans (2022) explores the dynamics of AI adoption in competitive markets, shedding light on how competition influences firms' decisions to adopt AI technologies. This work is complemented by Gans (2023), which contrasts AI adoption in competitive markets with its adoption in monopolistic contexts, offering a nuanced view of how market structures can shape the adoption of AI. McElheran et al. (2021) empirically analyze AI adoption across America. Their study identifies the who, what, and where of AI adoption, providing a detailed landscape of how AI technologies are being integrated across different industries and regions in the U.S. And many other scholars have explored the dynamics of algorithmic pricing, competition, and their impacts, including research by Calvano et al. (2019, 2020); Sanchez-Cartas and Katsamakas (2022); Brown and MacKay (2023); Normann and Sternberg (2023).

Our work mainly focus on the AI adoption by firms in a duopoly competitive environment offering multiple products. The core framework of our study is adapted from Johnson and Myatt (2003) on multi-product strategies. We analyze various AI adoption scenarios within a duopoly setting and examine their interplay with the pricing of AI services. This work enriches the AI adoption literature by providing insights from a multi-product perspective.

# 3 Model

In this section, we lay out a formal model that delineates the pricing strategy of an AI company (AC) alongside the product line configurations of two rival firms in the downstream market. We explore a scenario in which AC achieved a significant breakthrough in AI technology and endeavors to integrate it into a broader array of applications.

### 3.1 Supply

In the downstream market, suppose one firm (HC) has advantageous technological capabilities to manufacture higher-end products, while the other firm (MC), operates within medium-end market segments. However, it is important to note that there could exist overlap in the groups of their targeted consumers, meaning HC also has the capacity to cater to lower tier markets.

Traditionally, the technology landscape in the downstream market, excluding AI, is depicted as a quality ladder comprising $n+1$ levels. The production technology srves as entry barriers for market segments, with HC offering products with quality $q \in \{\underline{n}, \cdots, n\}$, and MC offering products $q \in \{1, \cdots, m\}$, where $m < n$.[1] The use of AI effectively improves a product's quality to the next level, so that a product with quality $q_j$ is as attractive to the consumers as which was originally of quality $q_{j+1}$. Notably, the scenario where HC produces the highest-tier product and implements AI technology corresponds to $q = n+1$.

Now, let us outline the timeline. Initially, AC sets a universal usage fee, $p_a$, for its service. Subsequently, both HC and MC simultaneously decide whether to adopt AI technology, with these decisions being public information. Following this, the two companies devise their production plans and compete in the downstream market. Both firms may prune some product lines, focusing only on a subset of vertically differentiated products. For firm $i \in \{HC, MC\}$, we denote its active product lines as $\{\underline{b}^i, \underline{b}^i + 1, \ldots, \bar{b}^i\}$. The lowest quality products available in the market are denoted by $\underline{b} = \min\{\underline{b}^H, \underline{b}^M\}$. Also, we denote by $\bar{b}$ the quality of the relatively higher baseline product provided by one of the firm $\max\{\underline{b}^H, \underline{b}^M\}$.

Throughout the remainder of the paper, we use subscripts to index "qualities" and superscripts "companies." The quality of a product at level $j$ produced by firm $i$ is represented as $q^i_j$. Note that $q_j = j$. Additionally, the unit cost of producing quality $j$ is denoted by $c_j$ without AI service and $c_{j-1} + p_a$ with AI service. We assume a symmetric cost structure between HC and MC in the initial downstream market before the introduction of AI.

In addition, to guarantee that competition in the downstream market occurs at multiple levels, we follow Johnson and Myatt (2003) to assume "decreasing returns to quality". Specifically, the increment on unit cost increases with quality level, $\Delta c_j = c_j - c_{j-1} < \Delta c_{j+1}$, $\forall j$, where $c_0$ is set to be 0.

### 3.2 Demand

There is a continuum of consumers with a measure of $\bar{\theta}$ *uniformly* distributed across the interval $[0, \bar{\theta}]$. This assumption of uniform distribution is adopted for technical convenience, facilitating a tractable analysis of firms' profits which can be written as a function of their production levels.

Consumers' preferences for product quality $q$ are represented by their respective types $\theta$, where $\theta \in [0, \bar{\theta}]$. The payoff function for a typical consumer of type $\theta$ purchasing a product of quality $q$

[1] This setting is not as general as in the original paper Johnson and Myatt (2003), where the gap between quality levels may differ. To facilitate the use of the "upgrade" approach, we make this regulatory assumption in order that: (1) The quality improving effect of AI is uniform across different quality levels; (2) AI improves the quality of a product to the next level, i.e., a product with quality $q_j$ will be upgraded to $q_{j+1}$. These assumptions streamline the analysis and offer a clearer insight into the implications of AI technology within the context of quality enhancement.

at price p is given by:

$$u_\theta(q,p) = \theta q - p.$$

Each consumer can buy at most one unit of a product. The measure of consumers purchasing product $j$ from company $i$ is denoted by $z^i_j$. This measure must satisfy the constraint: $\sum_{j=1}^{n+1}(z^H_j + z^M_j) = \bar{\theta}$. Here, $\mathbf{z}^i$ represents the vector of firm $i$'s production levels. Let $H(z)$ denote the threshold type above which there are consumers with measure $z$, corresponding to a downward sloping inverse demand function when the price of product is 1. Under the assumptions of this paper, $H(z) = \bar{\theta} - z$.

Given that we are analyzing a Cournot duopoly, the prices for products at different quality levels are determined by the production plans of both firms. Thus, the market price for a product with quality $j$ can be expressed as $p_j(\mathbf{z}^H, \mathbf{z}^M)$.

## 3.3 Profits

Absent AI technology, firm $i$'s profit is composed of the profits accrued from all market segments. For $i \in \{H, M\}$,

$$\pi^i = \sum_{j=\underline{b}^i}^{\bar{b}^i} z^i_j(p_j - c_j).$$

When firm $i$ adopts AI technology, it needs to pay $p_a$ to AC per vehicle it produces, then its profit is given by

$$\tilde{\pi}^i = \sum_{j=\underline{b}^i}^{\bar{b}^i} z^i_j(p_j - c_{j-1} - p_a).$$

The complication of this problem lies on that the profit from producing each quality is affected by the production at all quality levels. There is substitutability and complementarity between different product lines. To simplify the problem and provide a tractable characterization, we adopt the "upgrade" approach developed by Johnson and Myatt (2003). By this approach, we interpret each consumer's choice as being the combination of the initial decision on whether to buy the baseline product and the follow-up decision on which higher level should she updates the baseline product to. The amount of upgrade to quality $j$ and above is $Z^i_j = \sum_{k=j}^{n+1} z^i_k$. We will make intensive use of the notation $\{Z^i_j\}_{j=1}^{n+1}$ under the constraints that $Z^i_j \geq Z^i_{j+1}$, $\forall i, \forall j$. The total amount of production at quality level $j$ is $Z_j = Z^H_j + Z^M_j$. The benefit of using such an approach is that the determinants of the upgrades are relatively independent from upgrades at other levels, so that we can solve them in a similar way as we solve single-product duopoly competition.

After several transformations, the profit of firm $i$ can be equally written as a function of baseline product and upgrades.

$$\pi^i = \sum_{j=\underline{b}+1}^{\bar{b}^i} ((q_j - q_{j-1}) Z^i_j H(Z_j) - \Delta c_j Z^i_j) + (Z^i_{\underline{b}} q_{\underline{b}} H(Z_{\underline{b}}) - c_{\underline{b}} Z^i_{\underline{b}}). \tag{3.1}$$

If firm $i$ adopts AI technology and offers product with quality $j$ at cost $c_{j-1}+p_a$, its profit is given by

$$\tilde{\pi}^i = \sum_{j=\underline{b}+1}^{\bar{b}^i} ((q_j - q_{j-1})Z^i_j H(Z_j) - \Delta c_{j-1} Z^i_j) + (Z^i_{\underline{b}} q_{\underline{b}} H(Z_{\underline{b}}) - (c_{\underline{b}-1} + p_a) Z^i_{\underline{b}}). \tag{3.2}$$

### 3.4 Adoption of AI

| | | **H company** | |
|---|---|---|---|
| | | **Adopt** | **No** |
| **M company** | **Adopt** | $\tilde{\pi}^M_2$, $\tilde{\pi}^H_2$ | $\tilde{\pi}^M_3$, $\pi^H_3$ |
| | **No** | $\pi^M_4$, $\tilde{\pi}^H_4$ | $\pi^M_1$, $\pi^H_1$ |

Table 1: The Strategy Profiles of HC and MC Competition

Since HC and MC compete after they have chosen whether to adopt AI, in the second stage of the game, they are involved in a $2 \times 2$ simultaneously-move game. We refer Cases 1-4 to the strategy profiles ($No$, $No$), ($Adopt$, $Adopt$), ($Adopt$, $No$), and ($No$, $Adopt$), respectively. The strategy for solving this problem is that we will first analyze the production and market shares in the above four cases separately, and then we will turn to solve for the optimal pricing rule by the AI company.

The Nash equilibrium of this game depends on the AI price $p_a$, which has two effects on the downstream competition: (1) Adoption effect. When $p_a$ is low, both firms would like to adopt AI; but when $p_a$ rises, at most one firm will keep using AI, whereas its opponent would abandon AI technology for the sake of cost saving. (2) Production effect. Higher the price of AI, lower the baseline production the firm that adopts AI would offer. In contrast, strategic substitution between HC and MC entails expansion of the firm that does not adopt AI provided that AC sets a high price for its service.

In an attempt to maximize profits from selling AI service, AC needs to take into account competing firms' responses in the following stages. Indeed, AC would weigh a higher unit charge for AI against more firms' purchasing its service and a larger amount of products with AI being sold in the market.

## 4 Results

In this section, we present three results that shed light on how AI technology intersects with multi-product competition and pricing strategy of AI company. First, we will show that, in our framework, the profits of manufacturing firms operating in the downstream market exhibit a convenient functional forms depending on production levels, allowing us to relate firms' production schemes to its equilibrium profits. One rule of thumb is that if a firm expands the product lines of all its baseline product and upgrades, then we can ensure that its equilibrium profit also increases, without necessity to consider price and competition effects. The opposite direction also holds. Second, we explore the effect of AI usage price on firms' price, under each profile of strategies at the second stage of the game. It turns out that a higher price of AI leads to a lower profit for an AI-adopted firm, as it will reduce its upgrades at all levels. Third, we discuss the pattern of the transition of NE when AI usage price gradually increases. One notable feature is that when $p_a$ is relatively low, it is the unique NE that both firms adopt AI technology.

**Proposition 1.** *In any duopoly equilibrium, firm i's profit is the sum of squared production at the entire range of its product lines, regardless of whether AI is adopted in firm i's products.*

$$\pi^{i,*} = \sum_{j=1}^{\bar{b}^i} (Z_j^{i,*})^2, \qquad \tilde{\pi}^{i,*} = \sum_{j=1}^{\bar{b}^i} (Z_j^{i,*})^2. \tag{4.1}$$

*Because $Z_j^i = Z_{\underline{b}^i}^i$, for $i = H, M$ and $j < \underline{b}^i$, firm i's profit can be equivalently written as*

$$\pi^{i,*} = \sum_{\underline{b}^i+1}^{\bar{b}^i} (Z_j^{i,*})^2 + \underline{b}^i \cdot (Z_{\underline{b}^i}^{i,*})^2, \quad \tilde{\pi}^{i,*} = \sum_{\underline{b}^i+1}^{\bar{b}^i} (Z_j^{i,*})^2 + \underline{b}^i \cdot (Z_{\underline{b}^i}^{i,*})^2. \tag{4.2}$$

Proposition 1 provides a convenient form of equilibrium profit that is a quadratic function of upgrades at all quality levels. An immediate implication is that when a firm expands (contracts) all product lines in equilibrium, its profit will unambiguously increase (decrease). It turns out that $p_a$ does have monotonic effects on upgrades at all quality levels under any strategy profile of downstream firms. Then, a monotonicity relationship between the price for AI and profits follows.

**Proposition 2.** *Under any strategy profile, if firm i uses AI technology, then $\tilde{\pi}_i$ is weakly decreasing in $p_a$; if firm i does not use AI, then $\pi_i$ is weakly increasing in $p_a$.*

Next, we discuss the transition of NE when the value of $p_a$ rises from 0. On the one hand, it is intuitive that when $p_a$ is prohibitively high, AI service is too expensive to afford and both downstream firms would abandon AI technology. On the other hand, we will show that when $p_a$ is low, the unique equilibrium is that both firms adopt AI, since lagging in AI adoption would cause substantive losses of market shares and the capability to set a high price. In the middle range of $p_a$, however, one could see an NE that features only one firm adopting AI.

**Proposition 3.** *The pattern of transition of Nash equilibrium as $p_a$ rises is as follows.*

*(i) When $p_a$ is low, (Adopt, Adopt) is the unique Nash Equilibrium.*

*(ii) Above some threshold of $p_a$, the equilibria could be either (Adopt, No), (No, Adopt), or a mixed strategy equilibrium.*

*(iii) When $p_a$ is prohibitively high, (No, No) is the unique equilibrium.*

# 5 Numerical Example

We start by establishing our assumptions regarding the cost structure and quality differentiation among products, guided by the principle of "decreasing return to quality" as Johnson and Myatt (2003). This concept is mathematically represented as:

$$\frac{c_0}{q_0} < \frac{c_1 - c_0}{q_1 - q_0} < \frac{c_2 - c_1}{q_2 - q_1}.$$

This inequality suggests that as we move up the quality ladder, the marginal cost of quality increases. This setting enables us to model competition across multiple product lines between two automotive firms.

To illustrate these concepts, we consider a specific numerical example with predefined parameters: $c_0 = 1$, $c_1 = 3$, $c_2 = 30$, $q_0 = 1$, $q_1 = 2$, and $q_2 = 3$. Consequently, the marginal cost are computed as $\frac{c_0}{q_0} = 1$, $\frac{c_1 - c_0}{q_1 - q_0} = 2$, and $\frac{c_2 - c_1}{q_2 - q_1} = 27$. We also assume a uniform distribution of consumers across the preference spectrum $[0, \bar{\theta}]$, setting $\bar{\theta} = 28$. The consumer preference threshold, above which a consumer segment of size $Z$ exists, is defined as $H(Z) = 28 - Z$.

We then explore four distinct scenarios:

- Case I: Neither company adopts AI.
- Case II: Both companies adopt AI.
- Case III: The company MC adopts AI, while HC does not.
- Case IV: The company HC adopts AI, while MC does not.

For each scenario, we will calculate and compare the profits for HC and MC in the subsequent analyses, as depicted in Figures 1 and 2.

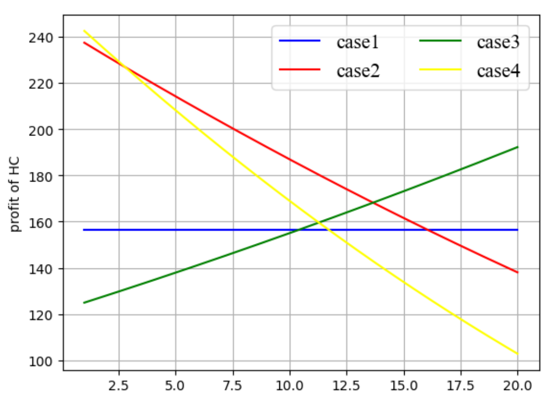


Figure 1: Profit of HC

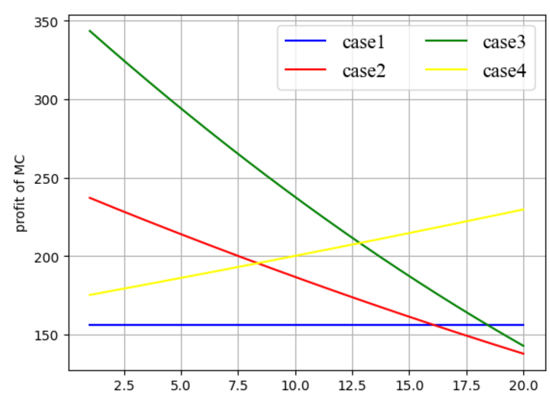


Figure 2: Profit of MC

Initially, at low values of $p_a$, both companies opt for AI integration, situating the Nash equilibrium at Case II, where both companies adopt AI, as demonstrated in Figure 1 and Figure 2. As

$p_a$ increases, the scenario shifts; MC's profit in Case IV surpasses its profit in Case II, leading to a new equilibrium at Case IV, where HC adopts AI and MC does not, as shown by the crossing of their respective profit lines in the figures.

As $p_a$ continues to rise, there comes a point where HC's profit in Case I is greater than in Case IV, indicating a disruption in the pure strategy Nash equilibrium. This leads to a mixed strategy equilibrium, illustrated in Figure 3, where HC adopts AI with probability $p$, and MC with probability $q$. The critical point occurs when $p_a$ is such that HC's profit in Case III equals its profit in Case II; here, $p$ equals $\frac{1}{2}$, and $q$ equals 1. At this juncture, MC will certainly adopt AI, whereas HC is indifferent between adopting and not adopting AI.

After this point, HC's profit in Case III exceeds its profit in Case II, so the Nash equilibrium becomes Case III, where MC adopts AI and HC does not.

Finally, when MC's profit in Case I exceeds its profit in Case III as $p_a$ further escalates, the equilibrium transitions to Case I. In this high $p_a$ environment, both companies refrain from adopting AI. The complete evolution of the Nash equilibrium from Case II to Case IV, then to a mixed strategy and to Case III, and ultimately to Case I is captured in Figure 3.

Figure 4 shows the profitability of an AI company, peaking at the aforementioned critical point where HC's profit in Case III is equivalent to that in Case II, with the optimal value of $p_a$ equal to 13.7, corresponding to an AI firm profit of 157.8. At this stage, MC is certain to adopt AI, while HC randomizes between two options. This outcome rationalizes the AI company's strategy to set $p_a$ at a level where MC is assured to embrace AI technology.

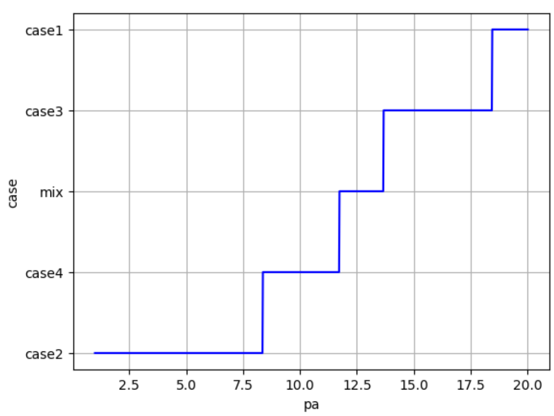


Figure 3: Nash equilibrium

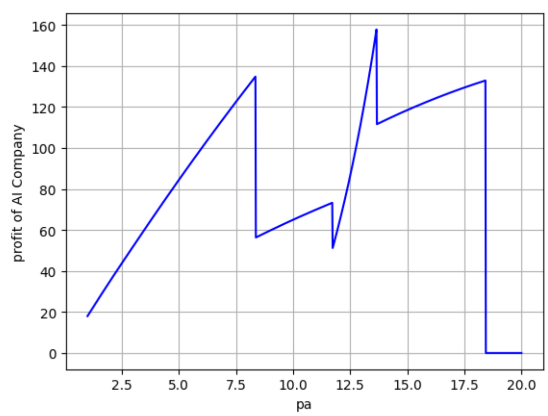


Figure 4: Profit of AC

Based on the data illustrated in Figures 5 and 6, we can draw several conclusions about the production strategies of the HC and MC companies under different Nash equilibrium cases.

When Case II represents the Nash equilibrium, we observe that as $p_a$ increases, the production of medium- and low-quality products by both HC and MC decreases, while the production of high-quality products by HC remains constant. This suggests that both HC and MC adjust their output in response to rising AI costs by reducing the production of low and medium quality products, but maintaining the production of high-quality products by HC.

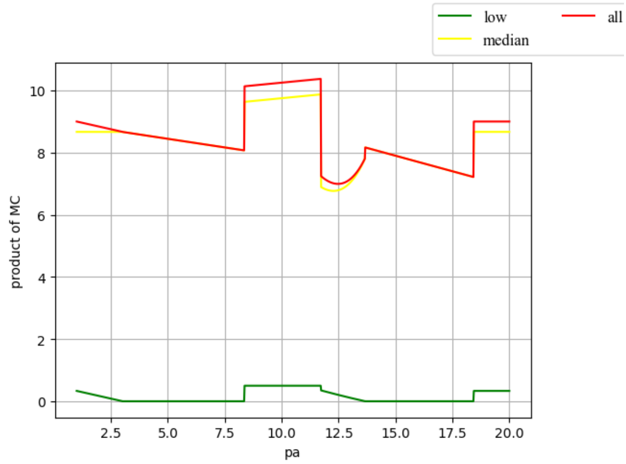


Figure 5: Product of MC

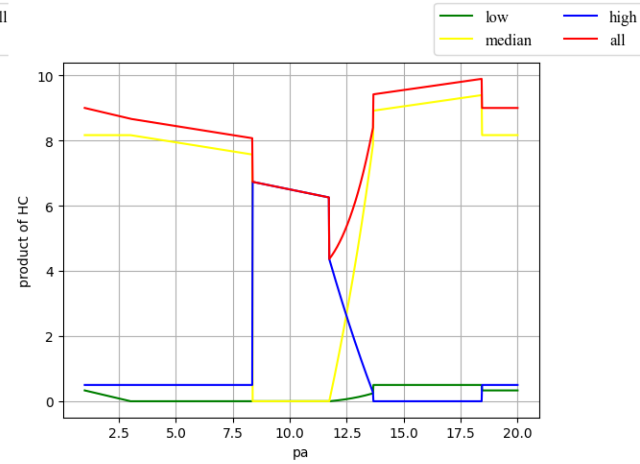


Figure 6: Product of HC

As we move to the Nash equilibrium represented by Case IV, HC appears to decrease its overall production but increases the output of high-quality products. This indicates a strategic shift towards focusing on high-value products when AI adoption becomes more costly. In contrast, MC increases the production of both medium- and low-quality products, which may reflect a strategy to capture a broader market segment that HC is moving away from due to their high-quality focus.

In the scenario of Case III being the Nash equilibrium, HC stops producing high-quality products altogether and instead boosts total output, suggesting a pivot towards more cost-effective product lines. MC responds by increasing the production of medium-quality products, yet its total output diminishes. This may reflect a resource reallocation to enhance the competitiveness of their medium-quality products.

These production changes underscore the dynamic interplay between product quality tiers and AI technology costs, with each company adjusting its output and quality focus to optimize profitability under varying conditions of AI integration costs.

# 6 Conclusion

In this paper, we investigate the adoption of AI technology by firms in a duopoly competitive environment with multiple products, particularly focusing on the optimal pricing strategy for AI companies when their technology could significantly impact the downstream market. The integration of AI technology into traditional industries represents a significant shift, offering enhanced product quality and operational efficiency. However, the adoption of AI is not uniform across the board, influenced by a variety of factors including pricing strategies of AI companies. Our analysis indicates that the pricing of AI services plays a crucial role in determining the extent of its integration within traditional sectors. A balanced approach to pricing can foster widespread adoption, benefiting both AI companies and traditional industries through improved competitiveness and product offerings. Conversely, disproportionate pricing may hinder the integration of AI, limiting its potential benefits. Therefore, AI companies need to carefully consider their pricing strategies, aiming to establish a price point that encourages adoption while optimizing their profit margins. This strategic decision-making is essential in ensuring that AI continues to be a driving force in the evolution of traditional industries, shaping the future landscape of market competition and product development.

# References


Agrawal, A. K., Gans, J. S., and Goldfarb, A. (2021). Ai adoption and system-wide change. Technical report, National Bureau of Economic Research.

Armstrong, M. and Vickers, J. (2018). Multiproduct pricing made simple. *Journal of Political Economy*, 126(4):1444–1471.

Brown, Z. Y. and MacKay, A. (2023). Competition in pricing algorithms. *American Economic Journal: Microeconomics*, 15(2):109–156.

Calvano, E., Calzolari, G., Denicolò, V., and Pastorello, S. (2019). Algorithmic pricing what implications for competition policy? *Review of industrial organization*, 55:155–171.

Calvano, E., Calzolari, G., Denicolo, V., and Pastorello, S. (2020). Artificial intelligence, algorithmic pricing, and collusion. *American Economic Review*, 110(10):3267–3297.

Champsaur, P. and Rochet, J.-C. (1989). Multiproduct duopolists. *Econometrica*, pages 533–557.

Chatterjee, S., Rana, N. P., Dwivedi, Y. K., and Baabdullah, A. M. (2021). Understanding ai adoption in manufacturing and production firms using an integrated tam-toe model. *Technological Forecasting and Social Change*, 170:120880.

Eckel, C. and Neary, J. P. (2010). Multi-product firms and flexible manufacturing in the global economy. *The Review of Economic Studies*, 77(1):188–217.

Gallego, G. and Wang, R. (2014). Multiproduct price optimization and competition under the nested logit model with product-differentiated price sensitivities. *Operations Research*, 62(2):450–461.

Gans, J. S. (2022). Ai adoption in a competitive market. Technical report, National Bureau of Economic Research.

Gans, J. S. (2023). Artificial intelligence adoption in a monopoly market. *Managerial and Decision Economics*, 44(2):1098–1106.

Johnson, J. P. (2017). Unplanned purchases and retail competition. *American Economic Review*, 107(3):931–965.

Johnson, J. P. and Myatt, D. P. (2003). Multiproduct quality competition: Fighting brands and product line pruning. *American Economic Review*, 93(3):748–774.

Johnson, J. P. and Myatt, D. P. (2006a). Multiproduct cournot oligopoly. *The RAND Journal of Economics*, 37(3):583–601.

Johnson, J. P. and Myatt, D. P. (2006b). On the simple economics of advertising, marketing, and product design. *American Economic Review*, 96(3):756–784.

Johnson, J. P. and Myatt, D. P. (2015). The properties of product line prices. *International Journal of Industrial Organization*, 43:182–188.

Johnson, J. P. and Myatt, D. P. (2018). The determinants of product lines. *The RAND Journal of Economics*, 49(3):541–573.

Johnson, J. P. and Rhodes, A. (2021). Multiproduct mergers and quality competition. *The RAND Journal of Economics*, 52(3):633–661.

Manova, K. and Yu, Z. (2017). Multi-product firms and product quality. *Journal of International Economics*, 109:116–137.

McElheran, K., Li, J. F., Brynjolfsson, E., Kroff, Z., Dinlersoz, E., Foster, L., and Zolas, N. (2021). Ai adoption in america: Who, what, and where. *Journal of Economics & Management Strategy*.

Nocke, V. and Schutz, N. (2018). Multiproduct-firm oligopoly: An aggregative games approach. *Econometrica*, 86(2):523–557.

Normann, H.-T. and Sternberg, M. (2023). Human-algorithm interaction: Algorithmic pricing in hybrid laboratory markets. *European Economic Review*, 152:104347.

Sanchez-Cartas, J. M. and Katsamakas, E. (2022). Artificial intelligence, algorithmic competition and market structures. *IEEE Access*, 10:10575–10584.

# Appendix A. Proofs

Proof of Eq.(3.1)

$$\begin{aligned}
\pi_i &= \sum_{j=\underline{b}^i}^{n} (p_j - c_j) z_j^i \\
&= \sum_{j=\underline{b}}^{n} (p_j - c_j) z_j^i && (z_j^i = 0 \ \text{ for all } j < \underline{b}^i)
\end{aligned}$$

$$
\begin{aligned}
&= \sum_{j=\underline{b}}^{n} \Big( \sum_{k=\underline{b}}^{j} ((p_k - c_k) - (p_{k-1} - c_{k-1})) z_j^i + (p_{\underline{b}} - c_{\underline{b}}) z_j^i \Big) \\
&= \sum_{j=\underline{b}}^{n} \sum_{k=\underline{b}}^{j} ((p_k - c_k) - (p_{k-1} - c_{k-1})) z_j^i + \sum_{j=\underline{b}}^{n} (p_{\underline{b}} - c_{\underline{b}}) z_j^i \\
&= \sum_{k=\underline{b}}^{n} ((p_k - c_k) - (p_{k-1} - c_{k-1})) \sum_{j=k}^{n} z_j^i + (p_{\underline{b}} - c_{\underline{b}}) \sum_{j=\underline{b}}^{n} z_j^i \qquad (\textstyle\sum_{i=1}^{n} \sum_{j=1}^{i} b_{ij} = \sum_{j=1}^{n} \sum_{i=j}^{n} b_{ij}) \\
&= \sum_{k=\underline{b}}^{n} (\Delta q_k H(Z_k) - \Delta c_k) Z_k^i + (q_{\underline{b}} H(Z_{\underline{b}}) - c_{\underline{b}}) Z_{\underline{b}}^i
\end{aligned}
$$

where the last equation follows from $\Delta p_k = \Delta q_k H(Z_k)$ and $p_{\underline{b}} = q_{\underline{b}} H(Z_{\underline{b}})$.

Proof of Eq.(3.2)

$$
\begin{aligned}
\tilde{\pi}_i &= \sum_{j=\underline{b}^i}^{n} (p_j - c_{j-1} - p_a) z_j^i \\
&= \sum_{j=\underline{b}}^{n} (p_j - c_{j-1} - p_a) z_j^i \qquad (z_j^i = 0 \text{ for all } j < \underline{b}^i) \\
&= \sum_{j=\underline{b}}^{n} \Big( \sum_{k=\underline{b}}^{j} ((p_k - c_{k-1}) - (p_{k-1} - c_{k-2})) z_j^i + (p_{\underline{b}} - c_{\underline{b}-1} - p_a) z_j^i \Big) \\
&= \sum_{j=\underline{b}}^{n} \sum_{k=\underline{b}}^{j} ((p_k - c_{k-1}) - (p_{k-1} - c_{k-2})) z_j^i + \sum_{j=\underline{b}}^{n} (p_{\underline{b}} - c_{\underline{b}-1} - p_a) z_j^i \\
&= \sum_{k=\underline{b}}^{n} ((p_k - c_{k-1}) - (p_{k-1} - c_{k-2})) \sum_{j=k}^{n} z_j^i + (p_{\underline{b}} - c_{\underline{b}-1} - p_a) \sum_{j=\underline{b}}^{n} z_j^i \\
&\qquad (\textstyle\sum_{i=1}^{n} \sum_{j=1}^{i} b_{ij} = \sum_{j=1}^{n} \sum_{i=j}^{n} b_{ij}) \\
&= \sum_{k=\underline{b}}^{n} (\Delta q_k H(Z_k) - \Delta c_{k-1}) Z_k^i + (q_{\underline{b}} H(Z_{\underline{b}}) - c_{\underline{b}-1} - p_a) Z_{\underline{b}}^i
\end{aligned}
$$

where the last equation follows from $\Delta p_k = \Delta q_k H(Z_k)$ and $p_{\underline{b}} = q_{\underline{b}} H(Z_{\underline{b}})$.

## A.1 Proof of Proposition 1

We only provide a proof for the case where firm $i$ does not adopt AI technology, while the case where firm $i$ adopts AI can be proved in the same manner.

*Case 1:* $\underline{b}^i = \underline{b}$

The F.O.C. with respect to $Z_j^i$ deriving from Eq. (3.1) are given as below:

For $j > \underline{b}^i$, FOC is $\bar{\theta} - 2Z_j^i - Z_j^{-i} - \Delta c_j = 0$, from which we can obtain the expression for the equilibrium production at quality level $j$, if this product line is not pruned:

$$Z_j^{i,*} = H(Z_j) - \Delta c_j. \tag{A.1}$$

For $j = \underline{b}^i$, FOC is $\underline{b}^i(\bar{\theta} - 2Z_{\underline{b}^i}^i - Z_{\underline{b}^i}^{-i}) = c_{\underline{b}^i}$. Then we have

$$Z_{\underline{b}^i}^{i,*} = H(Z_{\underline{b}}) - \frac{c_{\underline{b}^i}}{\underline{b}^i}. \tag{A.2}$$

Plugging Eq.(A.1)(A.2) to Eq (3.1), we will obtain that

$$\pi^{i,*} = \sum_{\underline{b}+1}^{\bar{b}^i} (Z_j^{i,*})^2 + \underline{b} \cdot (Z_{\underline{b}}^{i,*})^2 = \sum_{j=1}^{\bar{b}^i} (Z_j^{i,*})^2.$$

*Case 2: $\underline{b}^i > \underline{b}$*

$$\pi^i = \sum_{j=\underline{b}^i+1}^{\bar{b}^i} Z_j^i \cdot (H(Z_j) - \Delta c_j) + \sum_{\underline{b}+1}^{\underline{b}^i} Z_{\underline{b}^i}^i \cdot (H(Z_{\underline{b}^i}^i + Z_j^{-i}) - \Delta c_j) + \underline{b} Z_{\underline{b}^i}^i H(Z_{\underline{b}^i}^i + Z_{\underline{b}}^{-i}) - c_{\underline{b}} Z_{\underline{b}^i}^i.$$

From FOC with respect to $Z_j^i$, $j > \underline{b}^i$, $\bar{\theta} - 2Z_j^i - Z_j^{-i} - \Delta c_j = 0$, we have

$$Z_j^{i,*} = H(Z_j^*) - \Delta c_j. \tag{A.3}$$

From FOC with respect to $Z_{\underline{b}^i}^i$

$$\begin{aligned} &\sum_{j=\underline{b}+1}^{\underline{b}^i} (\bar{\theta} - 2Z_{\underline{b}^i}^i - Z_j^{-i} - \Delta c_j) + \underline{b} \cdot (\bar{\theta} - 2Z_{\underline{b}^i}^i - Z_{\underline{b}}^{-i}) - c_{\underline{b}} = 0 \\ \iff &\sum_{j=\underline{b}+1}^{\underline{b}^i} (H(Z_{\underline{b}^i}^i + Z_j^{-i}) - \Delta c_j) + \underline{b} \cdot H(Z_{\underline{b}^i}^i + Z_{\underline{b}}^{-i}) - c_{\underline{b}} = \underline{b}^i Z_{\underline{b}^i}^i. \end{aligned} \tag{A.4}$$

Plugging Eq. (A.3)(A.4) to Eq. (3.1), we have

$$\pi^{i,*} = \sum_{\underline{b}^i+1}^{\bar{b}^i} (Z_j^{i,*})^2 + \underline{b}^i \cdot (Z_{\underline{b}^i}^{i,*})^2 = \sum_{j=1}^{\bar{b}^i} (Z_j^{i,*})^2.$$

## A.2 Proof of Proposition 2

Let us discuss case by case. Since in Case 1 where no firm adopts AI, $\pi_1^i$ is independent of $p_a$, the result is trivial. In Case 2 where both firms adopt AI, the market segments for upgrades are unaffected, since $\Delta c_j$ stay the same. While for the baseline product, the firms' production, which could be equivalently calculated as in a single-product duopolistic market, declines due to higher cost $c_{\underline{b}-1} + p_a$.

### A.2.1 Case 2

When both firms adopt AI, they each produce quality-$k$ good at cost $c_{k-1}+p_a$. Then we can calculate their market shares as follows. Without loss of generality, we assume that Firm $i$ produces the lowest quality good ($\underline{b}=\underline{b}^i$) and that Firm $j$'s baseline product has the feature that $\bar{b}=\underline{b}^j$. When $\underline{b}^i=\underline{b}^j$, it is straightforward that baseline production $Z^i_{\underline{b}}$ and $Z^j_{\underline{b}}$ are both decreasing in $p_a$, with all upgrades unchanged. Therefore, the profits for both firms decline. Below, we only consider the case when $\bar{b}>\underline{b}$.

For $\underline{b}<k\le\bar{b}$, the FOC with respect to $Z^i_k$ is given by

$$\bar{\theta}-2Z^i_k-Z^j_{\bar{b}}=\Delta c_{k-1} \tag{A.5}$$

For the baseline product, the FOC with respect to $Z^i_{\underline{b}}$ is given by

$$\underline{b}(\bar{\theta}-2Z^i_{\underline{b}}-Z^j_{\bar{b}})=c_{\underline{b}-1}+p_a \tag{A.6}$$

For Firm $j$'s baseline product, the FOC with respect to $Z^j_{\bar{b}}$ is given by

$$\sum_{k=\underline{b}+1}^{\bar{b}}(\bar{\theta}-2Z^j_{\bar{b}}-Z^i_k-\Delta c_{k-1})+\underline{b}(\bar{\theta}-2Z^j_{\bar{b}}-Z^i_{\underline{b}})=c_{\underline{b}-1}+p_a \tag{A.7}$$

From Eq.(A.6)(A.5), we can obtain that $Z^i_k=\frac{1}{2}(\bar{\theta}-Z^j_{\bar{b}}-\Delta c_{k-1})$ and $Z^i_{\underline{b}}=\frac{1}{2}(\bar{\theta}-Z^j_{\bar{b}}-\frac{c_{\underline{b}-1}+p_a}{\underline{b}})$ and that

$$\bar{b}Z^j_{\bar{b}}=\sum_{k=\underline{b}+1}^{\bar{b}}Z^i_k+\underline{b}Z^i_{\underline{b}}. \tag{A.8}$$

Plugging into Eq. (A.7) and then substituting back into the expressions for $Z^i_k$, we have

$$\begin{cases} Z^j_{\bar{b}} &= \frac{1}{3}\bar{\theta}-\frac{1}{3\bar{b}}(c_{\bar{b}-1}+p_a) \\ Z^i_{\underline{b}} &= \frac{1}{3}\bar{\theta}+\frac{c_{\bar{b}-1}}{6\bar{b}}-\frac{c_{\underline{b}-1}}{2\underline{b}}-(\frac{1}{\underline{b}}-\frac{1}{3\bar{b}})p_a \\ Z^i_k &= \frac{1}{3}\bar{\theta}+\frac{1}{6\bar{b}}c_{\bar{b}-1}-\frac{1}{2}\Delta c_{k-1}+\frac{1}{6\bar{b}}p_a \end{cases} \tag{A.9}$$

Eq.(A.9) show that the firm producing higher quality goods is negatively affected by an increase in $p_a$, while the other firm benefits from such a rise.

Then we examine how the AI usage fee affects firms' profits in equilibrium. Initially, we know that for $k>\bar{b}$, firms' upgrades $Z^i_k$ are not affected by usage fee $p_a$, since only the cost difference between quality levels $\Delta c_{k-1}$ matters.

For Firm $j$, because $Z^j_{\bar{b}}$ decreases with $p_a$, from Eq.(3.2) higher usage fee $p_a$ gives rise to lower profit for Firm $j$.

For Firm $i$, it suffices for us to consider the change in $Z^i_k$, $k\le\bar{b}$.

$$\frac{\partial\pi^i}{\partial p_a}=\sum_{k=\underline{b}-1}^{\bar{b}}Z^i_k\frac{1}{3\bar{b}}-2\underline{b}(\frac{1}{\underline{b}}-\frac{1}{3\bar{b}})Z^i_{\underline{b}} \tag{A.10}$$

$$= \frac{1}{3\bar{b}}(\bar{b}Z^j_{\bar{b}} - \underline{b}Z^i_{\underline{b}}) - 2\underline{b}(\frac{1}{\underline{b}} - \frac{1}{3\bar{b}})Z^i_{\underline{b}} \qquad \text{(By Eq.(A.8))}$$

$$= \frac{1}{3}Z^j_{\bar{b}} - (\frac{1}{2} - \frac{\underline{b}}{3\bar{b}})Z^i_{\underline{b}} \tag{A.11}$$

A sufficient condition for $\pi^i$ to increase with $p_a$ is that $Z^j_{\bar{b}} > Z^i_{\underline{b}}$ and $\underline{b} > \frac{1}{6}\bar{b}$.

#### A.2.2 Case 3: only M adopts AI

When $\bar{b} = \underline{b}$, it is similar to the case of single product duopoly that the firm whose cost unilaterally increases will have less equilibrium production. Therefore, as $p_a$ rises, $\tilde{\pi}^M$ decreases while $\pi^H$ increases.

When $\underline{b}^M < \underline{b}^H$, the FOCs are given by

$$\begin{cases} \bar{\theta} - 2Z^M_k - Z^H_{\bar{b}} = \Delta c_{k-1} \qquad \text{for } k = \underline{b}+1, \dots, \bar{b} \\ \underline{b}(\bar{\theta} - 2Z^M_{\underline{b}} - Z^H_{\bar{b}}) = c_{\underline{b}-1} + p_a \\ \sum_{k=\underline{b}+1}^{\bar{b}}(\bar{\theta} - 2Z^H_{\bar{b}} - Z^M_k - \Delta c_k) + \underline{b}(\bar{\theta} - 2Z^H_{\bar{b}} - Z^M_{\underline{b}}) = c_{\underline{b}} \end{cases}$$

Rearranging these equations we have $\bar{b}Z^H_{\bar{b}} = \sum_{k=\underline{b}+1}^{\bar{b}} Z^M_k + \underline{b}Z^M_{\underline{b}} - \Delta c_{\bar{b}} + p_a$. Plugging the expressions of $Z^M_k$ and $Z^M_{\underline{b}}$ into this equation, we will have

$$Z^H_{\bar{b}} = \frac{2}{3\bar{b}}(\frac{1}{2}\bar{b}\bar{\theta} + \frac{1}{2}c_{\bar{b}-1} - c_{\bar{b}} + \frac{1}{2}p_a)$$

In equilibrium, $Z^H_{\bar{b}}$ increases with $p_a$ and $Z^M_k$, for $k = \underline{b}, \dots, \bar{b}$, decreases with $p_a$. From Proposition 1, their profits $\pi^H$ and $\tilde{\pi}^M$ are increasing and decreasing in $p_a$, respectively.

When $\underline{b}^M > \underline{b}^H$, the FOCs are given by

$$\begin{cases} \bar{\theta} - 2Z^H_k - Z^M_{\bar{b}} = \Delta c_k \qquad \text{for } k = \underline{b}+1, \dots, \bar{b} \\ \underline{b}(\bar{\theta} - 2Z^H_{\underline{b}} - Z^M_{\bar{b}}) = c_{\underline{b}} \\ \sum_{k=\underline{b}+1}^{\bar{b}}(\bar{\theta} - 2Z^M_{\bar{b}} - Z^H_k - \Delta c_{k-1}) + \underline{b}(\bar{\theta} - 2Z^M_{\bar{b}} - Z^H_{\underline{b}}) = c_{\underline{b}-1} + p_a \end{cases}$$

Rearranging these equations we have $\bar{b}Z^M_{\bar{b}} = \frac{1}{2}\sum_{k=\underline{b}+1}^{\bar{b}}(\bar{\theta} - Z^M_k - \Delta c_k) + \underline{b}Z^H_{\underline{b}}\Delta c_{\bar{b}} - p_a$. Plugging the expressions of $Z^M_k$ and $Z^M_{\underline{b}}$ into this equation, we will have

$$Z^M_{\bar{b}} = \frac{2}{3\bar{b}}(\frac{1}{2}\bar{b}\bar{\theta} + \frac{1}{2}c_{\bar{b}} - c_{\bar{b}-1} - p_a)$$

In equilibrium, $Z^M_{\bar{b}}$ decreases with $p_a$, and $Z^H_k$, for $k = \underline{b}, \dots, \bar{b}$ increases with $p_a$. From Proposition 1, their profits $\pi^H$ and $\tilde{\pi}^M$ are increasing and decreasing in $p_a$, respectively.

#### A.2.3 Case 4: only H adopts AI

This case is symmetric with case 3 and similar results follow. When $p_a$ rises, $\tilde{\pi}^H$ decreases while $\pi^M$ increases.

## A.3 Proof of Proposition 3

We want to show that when the use of AI is free, firms would unambiguously choose to upgrade from the perspective of competition. Formally, we demonstrate that when $p_a = 0$: (a) $\pi_1^H < \tilde{\pi}_4^H$; (b) $\pi_1^M < \tilde{\pi}_1^M$; (c) $\tilde{\pi}_4^M < \tilde{\pi}_2^M$; (d) $\pi_3^H < \tilde{\pi}_2^H$.

To make clear the comparison between different cases, we use $Y_j^i$ to denote the size of upgrade up to quality $j$ by firm $i$ if firm $i$ adopts AI. For example, in Case 3, $Y_j^M$ represents MC's production at quality $j$ or above and $Z_j^H$ represents HC's production at quality $j$ or above.

(a) $\pi_1^H < \tilde{\pi}_4^H$

First, we consider the case where $\underline{b}^H > \underline{b}^M$ and $\bar{b}^M > \underline{b}^H$. For $k > \bar{b}+1$, the upgrades increase due to lower marginal cost for production at different quality levels. So we only need to prove that $Y_{\bar{b}+1}^H > Z_{\bar{b}}^H$.

The baseline product without AI technology $Z_{\bar{b}}^H$ is calculated from

$$\begin{cases} \bar{\theta} - 2Z_k^M - Z_{\bar{b}}^H = \Delta c_k \qquad \text{for } k = \underline{b}+1, \dots, \bar{b} \\ \underline{b}(\bar{\theta} - 2Z_{\underline{b}}^M - Z_{\bar{b}}^H) = c_{\underline{b}} \\ \sum_{k=\underline{b}+1}^{\bar{b}} (\bar{\theta} - 2Z_{\bar{b}}^H - Z_k^M - \Delta c_k) + \underline{b}(\bar{\theta} - 2Z_{\bar{b}}^H - Z_{\underline{b}}^M) = c_{\underline{b}} \end{cases}$$

The result is that $Z_{\bar{b}}^H = \frac{1}{3}\bar{\theta} - \frac{1}{3\bar{b}} c_{\bar{b}}$.

In contrast, the baseline product with AI, $Y_{\bar{b}+1}^H$, is calculated from

$$\begin{cases} \bar{\theta} - 2Z_k^M - Y_{\bar{b}+1}^H = \Delta c_k \qquad \text{for } k = \underline{b}+1, \dots, \bar{b}+1 \\ \underline{b}(\bar{\theta} - 2Z_{\underline{b}}^M - Y_{\bar{b}+1}^H) = c_{\underline{b}} \\ \sum_{k=\underline{b}+1}^{\bar{b}+1} (\bar{\theta} - 2Y_{\bar{b}+1}^H - Z_k^M - \Delta c_{k-1}) + \underline{b}(\bar{\theta} - 2Y_{\bar{b}+1}^H - Z_{\underline{b}}^M) = c_{\underline{b}-1} \end{cases}$$

The result is that $Y_{\bar{b}+1}^H = \frac{1}{3}\bar{\theta} + \frac{2}{3(\bar{b}+1)}(\frac{1}{2}c_{\bar{b}+1} - c_{\bar{b}})$. It can be shown that $Y_{\bar{b}+1}^H > Z_{\bar{b}}^H$.

Second, we consider the case where $\underline{b}^H > \underline{b}^M$ and $\bar{b}^M \le \underline{b}^H$. That means Firm $H$ is a monopolistic producer at quality level $\bar{b}+1$.

The baseline product without AI technology $Z_{\bar{b}}^H$ is calculated from

$$\begin{cases} \bar{\theta} - 2Z_k^M - Z_{\bar{b}}^H = \Delta c_k \qquad \text{for } k = \underline{b}+1, \dots, \bar{b}^M \\ \underline{b}(\bar{\theta} - 2Z_{\underline{b}}^M - Z_{\bar{b}}^H) = c_{\underline{b}} \\ \sum_{k=\underline{b}+1}^{\bar{b}^M} (\bar{\theta} - 2Z_{\bar{b}}^H - Z_k^M - \Delta c_k) + \underline{b}(\bar{\theta} - 2Z_{\bar{b}}^H - Z_{\underline{b}}^M) = c_{\underline{b}} \end{cases}$$

The result is that $Z_{\bar{b}}^H = \frac{1}{3}\bar{\theta} - \frac{1}{3\bar{b}^M} c_{\bar{b}^M}$.

In contrast, the baseline product with AI, $Y_{\bar{b}+1}^H$, is calculated from

$$\begin{cases} \bar{\theta} - 2Z_k^M - Y_{\bar{b}+1}^H = \Delta c_k \qquad \text{for } k = \underline{b}+1, \dots, \bar{b}^M \\ \underline{b}(\bar{\theta} - 2Z_{\underline{b}}^M - Y_{\bar{b}+1}^H) = c_{\underline{b}} \\ \sum_{k=\underline{b}+1}^{\bar{b}^M} (\bar{\theta} - 2Y_{\bar{b}+1}^H - Z_k^M - \Delta c_{k-1}) + \underline{b}(\bar{\theta} - 2Y_{\bar{b}+1}^H - Z_{\underline{b}}^M) = c_{\underline{b}-1} \end{cases}$$

The result is that $Y^H_{\bar{b}+1} = \frac{1}{3}\bar{\theta} + \frac{1}{3\bar{b}^M} c_{\bar{b}^M} - \frac{2}{3\bar{b}^M} c_{\bar{b}^M-1}$. It can be shown that $Y^H_{\bar{b}+1} > Z^H_{\bar{b}}$.

Third, we consider the case where $\underline{b}^H = \underline{b} < \bar{b}^M$. Without AI technology, the production at different quality levels is calculated from

$$\begin{cases} \bar{\theta} - 2Z^H_k - Z^M_{\bar{b}} = \Delta c_k \qquad \text{for } k = \underline{b}+1, \ldots, \bar{b} \\ \underline{b}(\bar{\theta} - 2Z^H_{\underline{b}} - Z^M_{\bar{b}}) = c_{\underline{b}} \\ \sum_{k=\underline{b}+1}^{\bar{b}} (\bar{\theta} - 2Z^M_{\bar{b}} - Z^H_k - \Delta c_k) + \underline{b}(\bar{\theta} - 2Z^M_{\bar{b}} - Z^H_{\underline{b}}) = c_{\underline{b}} \end{cases}$$

The solutions are

$$\begin{cases} Z^M_{\bar{b}} &= \frac{1}{3}\bar{\theta} - \frac{1}{3\bar{b}} c_{\bar{b}} \\ Z^H_k &= \frac{1}{3}\bar{\theta} + \frac{1}{6\bar{b}} c_{\bar{b}} - \frac{1}{2}\Delta c_k \quad \text{for } k = \underline{b}+1, \ldots, \bar{b}. \\ Z^H_{\underline{b}} &= \frac{1}{3}\bar{\theta} + \frac{1}{6\bar{b}} c_{\bar{b}} - \frac{c_{\underline{b}}}{2\underline{b}} \end{cases}$$

After Firm H upgrades costlessly, its upgrades are denoted by $Y^H_k$, while Firm M's upgrades $X^M_k$. Then we solve the following system of equations:

$$\begin{cases} \bar{\theta} - 2Y^H_k - X^M_{\bar{b}} = \Delta c_{k-1} \qquad \text{for } k = \underline{b}+2, \ldots, \bar{b} \\ (\underline{b}+1)(\bar{\theta} - 2Y^H_{\underline{b}+1} - X^M_{\bar{b}}) = c_{\underline{b}} \\ \sum_{k=\underline{b}+2}^{\bar{b}} (\bar{\theta} - 2X^M_{\bar{b}} - Y^H_k - \Delta c_k) + (\underline{b}+1)(\bar{\theta} - 2X^M_{\bar{b}} - Y^H_{\underline{b}+1}) = c_{\underline{b}+1} \end{cases}$$

We have $X^M_{\bar{b}} = \frac{1}{3}\bar{\theta} - \frac{2}{3\bar{b}}(c_{\bar{b}} + c_{\underline{b}+1} - \frac{3}{2}c_{\underline{b}})$. It can be shown that $X^M_{\bar{b}} < Z^M_{\bar{b}}$, which means that Firm M contracts its production after Firm H's upgrading. Combining this inequality with the fact that $\Delta c_{k-1} < \Delta c_k$, we obtain that $Y^H_k > Z^H_k$, for $k = \underline{b}+1, \ldots, \bar{b}$. What remains to prove is that $Y^H_{\underline{b}+1} \geq Z^H_{\underline{b}}$.

$$Y^H_{\underline{b}+1} = \frac{1}{2}(\bar{\theta} - X^M_{\bar{b}} - \frac{c_{\underline{b}}}{\bar{b}+1}) > \frac{1}{2}(\bar{\theta} - Z^M_{\bar{b}} - \frac{c_{\underline{b}}}{\bar{b}}) = Z^H_{\underline{b}}$$

---

Results (ii) and (iii) are easy to see. Monotonicity of firms' profit functions of $p_a$ in Cases 2, 3, and 4 suggests that firms would deviate from *Adopt* when $p_a$ is above some threshold. In Section 4, the numerical example will illustrate the possibilities that $(Adopt, No)$ and $(No, Adopt)$ can be supported as NE. There could also exist mixed-strategy equilibria. In addition, there is $\bar{p}$ above which $(No, No)$ is the unique equilibrium. Then, it remains to prove result (i).

Because the profit functions are locally continuous in $p_a$, it suffices to show that at $p_a = 0$, $(Adopt, Adopt)$ is the unique equilibrium. Formally, let us demonstrate 4 claims: (a) $\pi^H_1 < \tilde{\pi}^H_4$; (b) $\pi^M_1 < \tilde{\pi}^M_1$; (c) $\tilde{\pi}^M_4 < \tilde{\pi}^M_2$; (d) $\pi^H_3 < \tilde{\pi}^H_2$.

(a) $\pi_1^H < \tilde{\pi}_4^H$

To make clear the comparison between different cases, we use $Y_j^i$ to denote the size of upgrade up to quality $j$ by firm $i$ if firm $i$ adopts AI. For example, in Case 3, $Y_j^M$ represents MC's production at quality $j$ or above and $Z_j^H$ represents HC's production at quality $j$ or above. In addition, the product lines for AI-using firm is denoted by $\{\underline{d}^i, \underline{d}^i + 1, \ldots, \bar{d}^i\}$.

Based on Proposition 1, we have

$$\pi_1^H = \sum_{j=\underline{b}^H+1}^{\bar{b}^H} (Z_j^H)^2 + \underline{b}^H \cdot (Z_{\underline{b}^H}^H)^2; \quad \tilde{\pi}_4^H = \sum_{j=\underline{d}^H+1}^{\bar{d}^H} (Y_j^H)^2 + \underline{d}^H \cdot (Y_{\underline{d}^H}^H)^2.$$

To prove the result, we are going to show that $Z_j^H < Y_j^H$ for all $j \le \bar{b}^H$. The discussion will be divided into two parts, depending on which firm operates in lower-end markets.

1. $\underline{b}^M \le \underline{b}^H$

   FOC w.r.t. $Z_{\underline{b}^H}^H$:

$$\sum_{j=\underline{b}^M+1}^{\underline{b}^H} (\bar{\theta} - 2Z_{\underline{b}^H}^H - Z_j^M) + \underline{b}^M \cdot (\bar{\theta} - 2Z_{\underline{b}^H}^H - Z_{\underline{b}^M}^M) = c_{\underline{b}^H}.$$

   FOC w.r.t. $Z_j^M$, $\underline{b}^M + 1 \le j \le \underline{b}^H$:

$$\bar{\theta} - 2Z_j^M - Z_{\underline{b}^H}^H = \Delta c_j.$$

   FOC w.r.t. $Z_{\underline{b}^M}^M$:

$$\underline{b}^M \cdot (\bar{\theta} - 2Z_{\underline{b}^M}^M - Z_{\underline{b}^H}^H) = c_{\underline{b}^M}.$$

   Solving this system of equations yields that

$$\begin{cases} Z_{\underline{b}^H}^H & = \frac{1}{3}(\bar{\theta} - \frac{c_{\underline{b}^H}}{\underline{b}^H}) \\ Z_{\underline{b}^H+1}^H & = \frac{1}{3}(\bar{\theta} - \Delta c_{\underline{b}^H+1}) \end{cases}$$

   However, if HC deviates to adopting AI, we can ensure that its all upgrades would expand because the cost for upgrade $j$ decreases from $\Delta c_j$ to $\Delta c_{j-1}$. At the meantime, the baseline product observes production at level

$$Y_{\underline{b}^H+1}^H = \frac{1}{3}(\bar{\theta} - \frac{2c_{\underline{b}^H} - c_{\underline{b}^H+1}}{\underline{b}^H + 1}).$$

   Since $Y_{\underline{b}^H+1}^H > Z_{\underline{b}^H}^H > Z_{\underline{b}^H+1}^H$, the baseline product line also expands as expected.

2. $\underline{b}^M > \underline{b}^H$
   FOC w.r.t. $Z_j^H$, for $\underline{b}^H < j < \underline{b}^M$:

$$\bar{\theta} - 2Z_j^H - Z_{\underline{b}^M}^M = \Delta c_j.$$

   FOC w.r.t. $Z_{\underline{b}^H}^H$:

$$\underline{b}^H \cdot (\bar{\theta} - 2Z_{\underline{b}^H}^H - Z_{\underline{b}^M}^M) = c_{\underline{b}^H}.$$

   FOC w.r.t. $Z_{\underline{b}^M}^M$:

$$\sum_{j=\underline{b}^H+1}^{\underline{b}^M} (\bar{\theta} - 2Z_{\underline{b}^M}^M - Z_j^H) + \underline{b}^H \cdot (\bar{\theta} - 2Z_{\underline{b}^M}^M - Z_{\underline{b}^H}^H) = c_{\underline{b}^M}.$$

   Solving this system of equations yields

$$\begin{cases} Z_{\underline{b}^M}^M & = \frac{1}{3}(\bar{\theta} - \frac{c_{\underline{b}^M}}{\underline{b}^M}) \\ Z_{\underline{b}^H}^H & = \frac{1}{2}(\bar{\theta} - Z_{\underline{b}^M}^M - \frac{c_{\underline{b}^H}}{\underline{b}^H}) \end{cases}$$

   However, if HC deviates to adopting AI, its upgrades increases ubiquitously as argued before; its base line product is given by

$$\begin{cases} \tilde{Z}_{\underline{b}^M}^M & = \frac{1}{3}(\bar{\theta} - \frac{c_{\underline{b}^M}}{\underline{b}^M}) \\ Y_{\underline{b}^H+1}^H & = \frac{1}{2}(\bar{\theta} - \tilde{Z}_{\underline{b}^M}^M - \frac{c_{\underline{b}^H}}{\underline{b}^H+1}) \end{cases}$$

   Since $Z_{\underline{b}^M}^M = \tilde{Z}_{\underline{b}^M}^M$, it is easy to see that $Y_{\underline{b}^H+1}^H > Z_{\underline{b}^H}^H$. Our result is achieved.

For results (b), (c), and (d), the proofs resemble that of (a), therefore we omit the details.